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A composite little Higgs model

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Abstract: We describe a natural UV complete theory with a composite little Higgs. Below a TeV we have the minimal Standard Model with a light Higgs, and an extra neutral scalar. At the TeV scale there are additional scalars, gauge bosons, and vector-like charge 2/3 quarks, whose couplings to the Higgs greatly reduce the UV sensitivity of the Higgs potential. Stabilization of the Higgs mass squared parameter, without finetuning, occurs due to a softly broken shift symmetry — the Higgs is a pseudo Nambu-Goldstone boson. Above the 10 TeV scale the theory has new strongly coupled interactions. A perturbatively renormalizable UV completion, with softly broken supersymmetry at 10 TeV is explicitly worked out. Our theory contains new particles which are odd under an exact "dark matter parity", $(-1)^{(2S+3B+L)}$. We argue that such a parity is likely to be a feature of many theories of new TeV scale physics. The lightest parity odd particle, or "LPOP", is most likely a neutral fermion, and may make a good dark matter candidate, with similar experimental signatures to the neutralino of the MSSM. We give a general effective field theory analysis of the calculation of corrections to precision electroweak observables.

Keywords: Beyond Standard Model.

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Contents

1. Introduction

A new mechanism for electroweak symmetry breaking, dubbed the "little Higgs" [1], was recently discovered via dimensional deconstruction [2, 3]. This mechanism has since been realized in various simple nonlinear sigma models [4–10]. In these theories, the Higgs is a pseudo Nambu-Goldstone boson, whose mass squared is protected against large radiative corrections by approximate nonlinearly realized global symmetries. These global symmetries are explicitly broken. However the symmetry breaking is sufficiently soft that the Higgs mass squared is at most logarithmically sensitive to the cutoff at one loop, provided the symmetry breaking terms satisfy a mild criterion: *no* single *term in the lagrangian breaks* all *the symmetry which is protecting the Higgs mass.* A collection of terms gives the Higgs its symmetry-breaking potential, gauge couplings, and Yukawa couplings. Such symmetry breaking may be thought of as being "nonlocal in theory space". Several new, weakly coupled particles are found at and possibly below a few TeV, which cancel the leading quadratic divergences in the Higgs mass in a manner reminiscent of softly broken

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supersymmetry. However, unlike supersymmetry, the cancellations occur between particles of the same statistics. The spectrum and phenomenology of little Higgs theories have been discussed in refs. [11 – 19].

In some of the simplest little Higgs theories, corrections to precision electroweak observables are comparable in size to one-loop Standard Model effects over much of the parameter space [20 – 22, 18, 23, 24], providing important constraints. It is, however, straightforward to find natural, simple, and experimentally viable little Higgs theories where the corrections are much smaller [18, 10, 23, 24, 8, 25]. This will be explicitly demonstrated in the model presented in this paper.

A pressing issue is to situate the little Higgs in a more complete theory with a higher cutoff. This is necessary in order to address in a compelling way the phenomenology of flavor changing neutral currents [26, 22], which is sensitive to physics beyond the 10 TeV scale. Further, unitarity constraints from little Higgs theories make a completion mandatory [27]. In this paper we embed a slightly altered version of the "littlest" [4] Higgs model into a UV complete theory. The model is experimentally viable, with acceptable precision electroweak corrections and no fine tuning. The Higgs is a composite of fermions interacting via strong dynamics at the 10 TeV scale. The quark and lepton masses can be generated without excessive flavor changing neutral currents. These interactions result from the Yukawa couplings of very heavy scalars, and can naturally arise in a theory with supersymmetry softly broken at 10 TeV. We stress that many other UV completions of little Higgs theories are possible, and the present model should by no means be taken as canonical. We simply wish to explore an explicit model thoroughly in order to better understand the theoretical and experimental implications of the little Higgs.

This model illustrates several advantages of composite little Higgs models as compared with the traditional approaches to electroweak symmetry breaking. In contrast to the Minimal Supersymmetric model (MSSM), there are no problems with fine tuning, lepton flavor violation, CP violation, or flavor changing neutral currents. The natural expectation for the masses of the new particles is well above the weak scale. The chief drawback of the model relative to the MSSM is the lack of a prediction for the weak angle.

This UV completion of the littlest Higgs model is quite similar to the Georgi-Kaplan Composite Higgs [28 – 32]. However unlike the Georgi-Kaplan models, in our model the hierarchy between the compositeness scale and the electroweak scale is not due to fine tuning of parameters.

We will describe this model from the bottom up, as a sequence of natural effective field theories. We start in section 2 with a description of the most important physics below 10 TeV. In section 3 we describe the strong dynamics which could lead to such an effective theory, and give a complete, renormalizable theory of the arbitrarily short distance physics. In section 4 we discuss "dark matter parity". This is a discrete symmetry which is an automatic consequence of baryon and lepton number conservation. In supersymmetry theories this symmetry is known as "R-parity", but the existence of such a symmetry is well motivated without supersymmetry. In section 5 we give an effective field theory analysis of the corrections to precision electroweak observables and the resulting bounds on the model. Section 6 addresses flavor physics and flavor changing neutral currents.

2. The SU(5)/ SO(5) little Higgs Model

The simplest of the little Higgs models, the "littlest Higgs", is a nonlinear sigma model whose target space is the coset space SU(5)/ SO(5) [4]. This theory describes the low energy interactions of 14 Nambu-Goldstone bosons (NGBs), with decay constant $f \sim 1-2$ TeV. The cutoff of this theory can be as high as $4\pi f \sim 10$ TeV, where the model becomes strongly coupled. The SU(5) symmetry is explicitly broken by $\mathcal{O}(1)$ gauge interactions and fermion couplings, leading to masses for most of the NGBs of order gf, while others get eaten by gauge bosons whose masses are also of order gf. A special subset of the NGBs, however, do not receive masses to leading order in the symmetry breaking terms, and are about a factor of $g^2/4\pi$ lighter than f. In the minimal model of ref. [4], this subset consisted only of a single Higgs doublet, dubbed the little Higgs. In the present model a neutral scalar may also remain light. At the TeV scale a small number of additional scalars, vector bosons and quarks cancel the one loop quadratic divergence in the Higgs mass without fine tuning or supersymmetry.

In order to make this paper self-contained, we describe the effective nonlinear sigma model in some detail here. We will describe the breaking as arising from a vacuum expectation value for a 5×5 symmetric matrix Φ, which transforms as $\Phi \to V\Phi V^T$ under SU(5). As we will see in the next section, it is straightforward to construct an explicit composite Higgs model where this Φ corresponds to a fermion bilinear. A vacuum expectation value for Φ proportional to the unit matrix breaks SU(5) $\to$ SO(5). For later convenience, we use an equivalent basis where the vacuum expectation value for the symmetric tensor points in the Σ_0 direction where Σ_0 is

$$\Sigma_0 = \begin{pmatrix} & & \mathbb{1} \\ & 1 & \\ \mathbb{1} & & \end{pmatrix}. \tag{2.1}$$

The unbroken SO(5) generators satisfy

$$T_a\Sigma_0 + \Sigma_0 T_a^T = 0 \tag{2.2}$$

while the broken generators obey

$$X_a\Sigma_0 - \Sigma_0 X_a^T = 0\,. \tag{2.3}$$

The Nambu-Goldstone bosons are fluctuations about this background in the broken directions $\Pi \equiv \pi^a X^a$, and can be parameterized by the non-linear sigma model field

$$\Sigma(x) = e^{i\Pi/f}\Sigma_0 e^{i\Pi^T/f} = e^{2i\Pi/f}\Sigma_0\,. \tag{2.4}$$

We now introduce the gauge and Yukawa interactions which explicitly break the global symmetry. In ref. [4], these were chosen to ensure an SU(3) global symmetry under which the little Higgs transformed nonlinearly, in the limit where any of the couplings were turned off. This required embedding the Standard Model $\mathrm{SU}(2)_w \times \mathrm{U}(1)_y$ gauge interaction into an $[\mathrm{SU}(2)\otimes \mathrm{U}(1)]^2$ gauge group, which was spontaneously broken to $\mathrm{SU}(2)_w \otimes \mathrm{U}(1)_y$ at the scale f. The extra U(1) and a factor of (1/5) in the charge matrix led to a rather light Z', which was constrained by Tevatron and precision electroweak corrections [20, 21, 18]. Due to the small size of the weak angle, we can eliminate the Z' and the associated constraints without increasing the finetuning of the theory. With a 10 TeV cutoff, naturalness does not require cancellation of its quadratically cutoff sensitive contribution to the Higgs mass squared from weak hypercharge gauge interactions. We therefore only introduce a single U(1). This simplification will make cancellation of gauge anomalies easy in the underlying composite model, as well as relaxing experimental constraints. We thus weakly gauge an $\mathrm{SU}(2)^2 \times \mathrm{U}(1)_y$ subgroup of the SU(5) global symmetry. The generators of the SU(2)'s are embedded into SU(5) as

$$Q_1^a = \begin{pmatrix} \sigma^a/2 & & \\ & & \\ & & \end{pmatrix} \qquad Q_2^a = \begin{pmatrix} & & \\ & & \\ & & -\sigma^{a*}/2 \end{pmatrix}. \tag{2.5}$$

while the generators of the U(1) are given by

$$Y = \mathrm{diag}(1, 1, 0, -1, -1)/2\,. \tag{2.6}$$

The electroweak SU(2) is generated by $Q_1^a + Q_2^a$, and is unbroken by Σ_0. The 14 NGBs have definite quantum numbers under the electroweak group. We write the NGB matrix as

$$\Pi = \begin{pmatrix} \frac{\eta}{\sqrt{40}}\mathbf{1} & \frac{h^T}{2} & \frac{\phi}{\sqrt{2}} \\ \frac{h^*}{2} & -\frac{2\eta}{\sqrt{10}} & \frac{\tilde{h}}{2} \\ \frac{\phi^\dagger}{\sqrt{2}} & \frac{\tilde{h}^\dagger}{2} & \frac{\eta}{\sqrt{40}}\mathbf{1} \end{pmatrix} \tag{2.7}$$

where h is the Higgs doublet, $h = (h^+, h^0)$, $\tilde{h} = (-h^0, h^+)$, η is a real neutral field and ϕ is an electroweak triplet carrying one unit of weak hypercharge, represented as a symmetric two by two matrix. We have ignored the three Goldstone bosons that are eaten in the Higgsing of $\mathrm{SU}(2)^2 \times \mathrm{U}(1) \to \mathrm{SU}(2) \times \mathrm{U}(1)$. In the following we will also neglect the η, which is an exact NGB to the order in which we are working. In order to avoid phenomenological problems from a massless NGB, which, for instance, is constrained by rare kaon decays, we can add small symmetry breaking terms to the potential in order to give the η a small mass, without significantly affecting the discussion of the little Higgs.

The effective theory at the scale f has a tree-level lagrangian given by

$$\mathcal{L} = \mathcal{L}_K + \mathcal{L}_t + \mathcal{L}_\psi\,. \tag{2.8}$$

Here $\mathcal{L}_K$ contains the kinetic terms for all the fields; $\mathcal{L}_t$ generates the top Yukawa coupling; and $\mathcal{L}_\psi$ generates the remaining small Yukawa couplings.

The kinetic terms for the fermions and gauge fields are conventional. The leading two-derivative term for the non-linear sigma model is

$$\frac{f^2}{4}\,\mathrm{tr}|D_\mu \Sigma|^2 \tag{2.9}$$

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		$SU(3)_c$	$SU(2)'$	$SU(2)$	$U(1)_Y$
X	p	3	1	2	7/6
	$\tilde{t}$	3	1	1	2/3
	$\tilde{q}$	3	2	1	1/6
$\bar{X}$	$\bar{\tilde{q}}$	$\bar{3}$	1	2	−1/6
	$\bar{\tilde{t}}$	$\bar{3}$	1	1	−2/3
	$\bar{p}$	$\bar{3}$	2	1	−7/6

Table 1: Charged 2/3 vector-like quark content.

where the covariant derivative of Σ is given by

$$D\Sigma = \partial\Sigma - \sum_j \left\{ig_j W_j^a (Q_j^a \Sigma + \Sigma Q_j^{aT}) + ig'_B (Y\Sigma + \Sigma Y^T)\right\}. \tag{2.10}$$

The g_i, g' are the couplings of the $SU(2)^2 \times U(1)_y$ groups.

We now turn to the important top Yukawa generating sector. The largest radiative corrections to the Higgs potential come from the top coupling. In the minimal Standard Model, the top Yukawa coupling leads to a large, negative, quadratically divergent mass term at one loop. In theories without one loop quadratic divergences, such as softly broken supersymmetry and little Higgs theories, the dominant correction to the quadratic terms in the Higgs potential come from the top sector, are negative, and are proportional to the scale of new physics squared. We therefore expect new particles associated with this sector to be lighter than a few TeV. In ref. [4] the minimal such sector was shown to contain a pair of colored left handed Weyl fermions $\tilde{t}, \tilde{t}^c$, in addition to the usual third-family weak doublet $Q = (t, b')$ and weak singlet $\bar{U}$. In the present model, we describe a possibility which arises more naturally in composite model building. We expect the SU(5) symmetry to arise as an accidental symmetry of the dynamics of a strongly coupled theory, analogous to the $SU(3) \times SU(3)$ chiral symmetry of QCD. Composite fermions will naturally couple to the composite bosons. We therefore introduce new fermions $X, \bar{X}$ transforming as $(5, 3)$ and $(5, \bar{3})$ respectively under $SU(5) \times SU(3)_c$, which gain mass from the $SU(5)/SO(5)$ symmetry breaking. These couple to the Σ field in an SU(5) symmetric fashion, and gain mass of order a TeV. The top mass will arise by mixing Q and $\bar{U}$ with composite quarks of the same quantum numbers, in a manner similar to Froggatt-Nielsen models of flavor [33] and the top see-saw [34, 35]. Explicitly, the fields $X, \bar{X}$, contain components $\tilde{q}, \tilde{t}, p, \bar{p}, \bar{\tilde{t}}, \bar{\tilde{q}}$, transforming under $SU(3)_c \times SU(2)' \times SU(2) \times U(1)_y$ as in table 1. We break the SU(5) symmetry only through explicit fermion mass terms connecting the Q and $\bar{T}$ to the components of $X, \bar{X}$ with the appropriate quantum numbers. The top Yukawa coupling arises from

$$\mathcal{L}_t = \lambda_1 f \bar{X} \Sigma^\dagger X + \lambda_2 f \bar{\tilde{q}} Q + \lambda_3 f \bar{T} \tilde{t} + \text{h.c.} \tag{2.11}$$

Because all three terms are needed to entirely break all the symmetry protecting the little Higgs mass, this form of symmetry breaking is soft enough to avoid quadratic or logarithmic divergences at one loop, or quadratic divergences at two loops. Thus at one loop, the largest radiative corrections to the Higgs potential are insensitive to the UV and computable in the low energy effective theory.

To see that $\mathcal{L}_t$ generates a top Yukawa coupling we expand $\mathcal{L}_t$ to first order in the Higgs h:

$$\mathcal{L}_t \supset \lambda_1 \bar{\tilde{t}} \tilde{q} h + f(\lambda_1 \bar{\tilde{t}} + \lambda_3 \bar{T})\tilde{t} + f\bar{\tilde{q}}(\lambda_1 \tilde{q} + \lambda_2 Q) + \cdots . \tag{2.12}$$

Clearly $\tilde{t}$ marries the linear combination $(\lambda_1 \bar{\tilde{t}} + \lambda_3 \bar{T})/(\lambda_1^2 + \lambda_3^2)^{1/2}$ to become massive, $\bar{\tilde{q}}$ marries the linear combination $(\lambda_1 \tilde{q} + \lambda_2 Q)/(\lambda_1^2 + \lambda_2^2)^{1/2}$, and p pairs up with $\bar{p}$. We can integrate out these heavy quarks. The remaining light combinations are q_3, the left handed doublet comprised mostly of top and bottom,

$$q_3 \equiv \frac{(\lambda_2 \tilde{q} - \lambda_1 Q)}{\sqrt{\lambda_1^2 + \lambda_2^2}} , \tag{2.13}$$

and $\bar{t}$, the left handed quark which is mostly anti-top,

$$\bar{t} \equiv \frac{(\lambda_3 \bar{\tilde{t}} - \lambda_1 \bar{T})}{\sqrt{\lambda_1^2 + \lambda_3^2}} , \tag{2.14}$$

with a Yukawa coupling to the little Higgs

$$\lambda_t \, h \bar{t} q_3 + \text{h.c.} \qquad \text{where} \qquad \lambda_t = \frac{\lambda_1 \lambda_2 \lambda_3}{\sqrt{\lambda_1^2 + \lambda_2^2}\sqrt{\lambda_1^2 + \lambda_3^2}} . \tag{2.15}$$

Finally, the interactions in $\mathcal{L}_\psi$ encode the remaining Yukawa couplings of the Standard Model. These couplings are explicitly SU(5) breaking but small enough so that the 1-loop quadratically divergent contributions to the Higgs mass they induce are negligible with a cutoff $\Lambda \sim 10\,\text{TeV}$. Note that since there may be additional fermions at the cutoff which cancel the anomalies involving the broken subgroup we need only to insist theStandard Model anomalies cancel in the effective theory at the TeV scale.

We now turn to a detailed discussion of loop effects in this effective theory, which give the Higgs an electroweak symmetry breaking potential.

2.1 The effective potential and electroweak symmetry breaking

At tree level the orientation of our Σ field is undetermined, and all the NGBs, including the little Higgs, are massless. We will compute the NGB effective potential at one loop order. Of course, our non-renormalizable effective theory is incomplete, and we will need to add new interactions (counterterms) in order to account for the cutoff sensitivity introduced by radiative corrections. We follow a standard chiral lagrangian analysis, including all operators consistent with the symmetries of the theory with coefficients assumed to be of the order determined by naïve dimensional analysis [36 − 38], that is, of similar size to the radiative corrections computed from the lowest order terms with cutoff $\Lambda = 4\pi f$.

The largest corrections come from the gauge sector, due to 1-loop quadratic divergences. Remarkably, the one loop quadratically divergent terms from the SU(2) gauge interactions do not contribute to the little Higgs mass squared. The gauge divergence is proportional to

$$\frac{\Lambda^2}{16\pi^2}\,\mathrm{tr}\, M_V^2(\Sigma) \tag{2.16}$$

where $M^2(\Sigma)$ is the gauge boson mass matrix in a background Σ. $M_V^2(\Sigma)$ can be read off from the covariant derivative for Σ (2.10), giving a potential

$$c g_j^2 f^4 \sum_a \mathrm{tr}\left[(Q_j^a\Sigma)(Q_j^a\Sigma)^*\right] + c g'^2 f^4\, \mathrm{tr}\left[(Y\Sigma)(Y\Sigma)^*\right]. \tag{2.17}$$

Here c is an $\mathcal{O}(1)$ constant which is sensitive to the UV physics at the scale Λ. Note that at second order in the gauge couplings and momenta (2.17) is the unique gauge invariant term transforming properly under the global SU(5) symmetry. This potential is similar to that generated by electromagnetic interactions in the pion chiral lagrangian, which shift the masses of $\pi^\pm$ from that of the π^0 [39], by an amount which is quadratically sensitive to the physics of the GeV scale. In analogy to the QCD chiral lagrangian, we assume that c is positive. This implies that the gauge interactions prefer the alignment Σ_0 where the electroweak group remains unbroken.

To quadratic order in ϕ and quartic order in h, the potential from (2.17) is

$$\begin{aligned} &c g_1^2 f^2 \left|\phi_{ij} - \frac{i}{2f}(h_i h_j + h_j h_i)\right|^2 + c g_2^2 f^2 \left|\phi_{ij} + \frac{i}{2f}(h_i h_j + h_j h_i)\right|^2 + \\ &+ c g'^2 \left(f^2(2h_i^* h_i + 4\phi_{ij}^*\phi_{ij}) - \frac{1}{3}(h^\dagger h)^2\right). \end{aligned} \tag{2.18}$$

The term (2.18) gives the triplet a positive mass squared of

$$m_\phi^2 = c(g_1^2 + g_2^2 + 4g'^2) f^2\,. \tag{2.19}$$

The little Higgs doublet, however, only receives mass at this order from the $\mathrm{U}(1)_Y$ interactions, because the $\mathrm{SU}(2)_{1,2}$ gauge interactions each leave an SU(3) symmetry intact, under which the little Higgs transforms nonlinearly [4]. The SU(2) interactions do, however, lead to an effective quartic interaction term in the little Higgs potential, as well as interaction with the ϕ triplet. After the Higgs triplet is integrated out, the resulting quartic coupling for the little Higgs is

$$\lambda = c\frac{4g_1^2 g_2^2 + \frac{11}{3}(g_1^2+g_2^2)g'^2 - \frac{4}{3}g'^4}{g_1^2+g_2^2+4g'^2}\,. \tag{2.20}$$

Note that a miracle seems to occur: the SU(2) interactions do not lead to a mass squared for the little Higgs at this order, but do give a quartic term in the Higgs potential which is of order g^2 when c is of order 1.

The remaining part of the vector boson contribution to the Coleman-Weinberg potential is

$$\frac{3}{64\pi^2}\,\mathrm{tr}\, M_V^4(\Sigma)\log\frac{M_V^2(\Sigma)}{\Lambda^2}\,. \tag{2.21}$$

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	p_t	$\tilde{t}$	$\tilde{q}_t$	Q_t
$\bar{p}_t$	$\lambda_1 f \cos^2\theta$	$\lambda_1 f \frac{i}{\sqrt{2}} \sin 2\theta$	$-\lambda_1 f \sin^2\theta$	0
$\bar{\tilde{t}}$	$\lambda_1 f \frac{i}{\sqrt{2}} \sin 2\theta$	$\lambda_1 f \cos 2\theta$	$\lambda_1 f \frac{i}{\sqrt{2}} \sin 2\theta$	0
$\bar{\tilde{q}}_t$	$-\lambda_1 f \sin^2\theta$	$\lambda_1 f \frac{i}{\sqrt{2}} \sin 2\theta$	$\lambda_1 f \cos^2\theta$	$\lambda_2 f$
$\bar{T}$	0	$\lambda_3 f$	0	0

Table 2: Charged 2/3 quark mass matrix.

This gives a logarithmically enhanced positive Higgs mass squared from the SU(2) interactions

$$\delta m_h^2 = \frac{9g^2 {M'_W}^2}{64\pi^2} \log \frac{\Lambda^2}{{M'_W}^2} \tag{2.22}$$

where M'_W is the mass of the heavy SU(2) triplet of gauge bosons. This contribution can be less than of order 10 times the required value, as is needed to avoid more than 10% fine-tuning, provided the W' is lighter than of order 6 TeV. There is a similar Coleman-Weinberg potential from the scalar self-interactions in eq. (2.17) which also give logarithmically enhanced positive contributions to the Higgs mass squared:

$$\delta m_h^2 = \frac{\lambda}{16\pi^2} M_\phi^2 \log \frac{\Lambda^2}{M_\phi^2} \tag{2.23}$$

where M_ϕ is the triplet scalar mass.

In this theory the top drives electroweak symmetry breaking, due to a similar quantum correction to the one which gives radiative electroweak symmetry breaking in the MSSM. Unlike in the MSSM, however, the one loop correction from the top sector has no contribution proportional to the logarithm of the cutoff, allowing for a somewhat higher new physics scale. The quark loop contribution to the one loop potential is

$$-\frac{3}{16\pi^2} \operatorname{tr} \left(M_f(\Sigma) M_f^\dagger(\Sigma) \right)^2 \log \frac{M_f(\Sigma) M_f^\dagger(\Sigma)}{\Lambda^2} \tag{2.24}$$

where $M_f(\Sigma)$ is the fermion mass matrix in a background Σ. We can neglect the contributions of the light fermions to this potential, and only consider the effects of the heavy charge 2/3 quarks contained in $\tilde{t}, \bar{\tilde{t}}, \tilde{q}_t, \bar{\tilde{q}}_t, p_t, \bar{p}_t, Q_t$ and $\bar{T}$. Here $Q_t, \tilde{q}_t, \bar{\tilde{q}}_t, p_t, \bar{p}_t$ denote the charge 2/3 components of the respective weak doublets. The charge 2/3 quark mass matrix is given in table 2, where $\theta = \langle h \rangle / (\sqrt{2} f)$. Note that

$$\frac{\partial}{\partial\theta} \operatorname{Tr} M^\dagger M = 0 \tag{2.25}$$

and

$$\frac{\partial}{\partial\theta} \operatorname{Tr} (M^\dagger M)^2 = 0 \tag{2.26}$$

which guarantees cutoff insensitivity of the one loop radiative corrections to the Higgs

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potential from this sector. Besides the top which has mass $\lambda_t\langle h\rangle$, there are three heavy quarks, of mass

$$
\begin{aligned}
M_1 &= \lambda_1 f \\
M_2 &= \left(a^2 + \frac{\lambda_t^2\langle h\rangle^2 b^2}{a^2-b^2} - \frac{\lambda_t^4\langle h\rangle^4(a^4-a^2b^2+b^4)}{(a^2-b^2)^3} + \mathcal{O}(\langle h\rangle^6)\right)^{1/2} \\
M_3 &= \left(b^2 - \frac{\lambda_t^2\langle h\rangle^2 a^2}{a^2-b^2} + \frac{\lambda_t^4\langle h\rangle^4(a^4-a^2b^2+b^4)}{(a^2-b^2)^3} + \mathcal{O}(\langle h\rangle^6)\right)^{1/2}
\end{aligned}
\tag{2.27}
$$

where

$$
\begin{aligned}
a^2 &= (\lambda_1^2+\lambda_2^2)f^2 \\
b^2 &= (\lambda_1^2+\lambda_3^2)f^2\,.
\end{aligned}
\tag{2.28}
$$

We denote these three heavy charge 2/3 quarks as the the t', t'', t''', respectively. Note that if mixing terms of order $\langle h\rangle/f$ are neglected, these quarks have vector-like Standard Model gauge quantum numbers (3,2,7/6), (3,1,2/3), and (3,2,1/6) respectively. Including the top and t'', t''' in equation (2.24) gives a contribution to the little Higgs effective potential

$$
\begin{aligned}
\delta V(h)_{\text{eff}} =& -\frac{3\lambda_t^2 h^\dagger h}{8\pi^2}\frac{a^2b^2}{a^2-b^2}\log\frac{a^2}{b^2} + \\
&+\frac{3\lambda_t^4(h^\dagger h)^2}{16\pi^2}\left(\frac{(a^2+b^2)\Big((3a^4+3b^4-4b^2a^2)\log\left(\frac{a^2}{b^2}\right) - (a^4-b^4)\Big)}{(a^2-b^2)^3} + 2\log\left(\frac{ab}{h^2}\right)\right) + \\
&+\mathcal{O}(h^6)\,.
\end{aligned}
\tag{2.29}
$$

Note that the contribution to the mass squared is negative while the contribution to the quartic term is positive, and numerically non-negligible. The negative contribution to the quadratic term from the top sector is the origin of electroweak symmetry breaking. Realistic electroweak symmetry breaking is possible for, e.g. $M_2 \sim M_3 \sim 2.8\,\text{TeV}$, $M_1 \sim 2\,\text{TeV}$, $c = 1$, $g_1/g_2 = 5$, $\Lambda \sim 10\,\text{TeV}$ and $f = 1\,\text{TeV}$. For these parameters, the physical Higgs particle mass is about 480 GeV,[1] and the quadratic terms in the Higgs potential from, respectively, gauge loops and fermion loops are about $+(420\,\text{GeV})^2$ and $-(550\,\text{GeV})^2$. Note this represents about 40% cancellation in the quadratic terms, which is not fine tuned. As f (and the masses of the heavy particles) is increased, the amount of fine-tuning necessary to obtain the correct electroweak symmetry breaking scale will scale as $1/f^2$, and models with $f \gtrsim 2\,\text{TeV}$ will typically be more than 10% fine-tuned.

3. UV completion: the little Higgs as a composite Higgs

The SU(5) $\to$ SO(5) symmetry breaking pattern can easily arise from fermion condensation through technicolor-like strong interactions, as in an old Composite Higgs model of Dugan,

[1] At this point the reader may be concerned about a discrepancy between such a heavy Higgs and precision electroweak bounds. Note that a Higgs of up to 500 GeV can be consistent with data when there are other nonstandard corrections [40, 41]. Note also that a lighter Higgs may easily be accommodated with smaller value for c, and an additional positive contribution to the Higgs mass squared from some additional symmetry breaking source.

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Georgi and Kaplan [32]. In this section we describe such a UV completion of the nonlinear sigma model model into a composite Higgs model. The SU(5)/ SO(5) symmetry breaking pattern arises from condensation of a new set of fermions, called Ultrafermions, which transform in a real representation of a new gauge group, called Ultracolor. For concreteness, and because it leads to an elegant mechanism for giving the top a large mass, we will take Ultracolor to be an SO(7) gauge group. The generation of fermion masses will require four fermion operators. It is conceivable that such operators might be nearly marginal in a strongly coupled, nearly conformal theory, in which case a theory with such operators might be UV complete up to very large energies. One might also consider generating them from an extended, Higgsed gauge group, as in extended ultracolor. In the present paper, however, we will assume the needed operators are generated by heavy scalar exchange. Such scalars are natural in a softly broken supersymmetric theory with a high supersymmetry breaking scale.

In this section we describe such a little Higgs theory from the top down. This theory is UV complete up to a scale as high as the Planck scale. At high energy we have a supersymmetric theory with soft supersymmetry breaking at 10 TeV. Although the UV physics is supersymmetric, the supersymmetry is irrelevant for phenomenology, as all superpartners are beyond direct experimental reach for the foreseeable future, and are too heavy to lead to indirect signals such as flavor changing neutral currents or lepton flavor violation.

3.1 Matter content above 10 TeV

We begin our description with a complete list of all the matter superfields in the theory and their gauge transformations under SO(7) ⊗ SU(3) ⊗ SU(2) ⊗ SU(2) ⊗ U(1) (see table 3). Here $i = 1, 2, 3$ is a generational index. $\mathrm{SU}(3)_c$ and $\mathrm{U}(1)_Y$ are color and weak hypercharge respectively. Weak isospin is the diagonal subgroup of the two SU(2)'s, and SO(7) is the ultracolor group responsible for the fermion condensate which dynamically breaks SU(5) → SO(5). The fields Y and $\bar{Y}$ are included to cancel $\mathrm{SU}(2)^2\,\mathrm{U}(1)$ and $\mathrm{SU}(2)'^2\,\mathrm{U}(1)$ anomalies, so that the theory is free of all gauge anomalies.

	SO(7)	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)'$	SU(2)	$\mathrm{U}(1)_Y$
L_i	1	1	1	2	$-1/2$
Q_i	1	3	1	2	$1/6$
$\bar{U}_i$	1	$\bar{3}$	1	1	$-2/3$
$\bar{D}_i$	1	$\bar{3}$	1	1	$1/3$
$\bar{E}_i$	1	1	1	1	1
$\Phi_{\bar{3}}$	7	$\bar{3}$	1	1	$-2/3$
Φ_3	7	3	1	1	$2/3$
$\Phi_{2'}$	7	1	2	1	$-1/2$
Φ_2	7	1	1	2	$1/2$
Φ_0	7	1	1	1	0
$\bar{Y}$	1	$\bar{3}$	1	2	$-7/6$
Y	1	3	2	1	$7/6$

Table 3: Chiral superfields in the theory above 10 TeV.

The approximate SU(5) global symmetry of the littlest Higgs nonlinear sigma model acts on the fields Φ_2, $\Phi_{2'}$, and Φ_0. The fermion components of these fields will bind to form the composite Higgs. The SU(5) symmetry is explicitly broken by the $\mathrm{SU}(2)' \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y$ gauge interactions and by the superpotential interactions below.

3.2 The UV lagrangian

The lagrangian of the theory above the 10 TeV scale contains the gauge interactions and the following superpotential interactions:

$$W = h_q Q_3 \Phi_{\bar{3}} \Phi_2 + h_{\bar{u}} \bar{U}_3 \Phi_3 \Phi_0 + h'_s Y \Phi_{\bar{3}} \Phi_{2'} + h_s \bar{Y} \Phi_3 \Phi_2 + m_3 \Phi_{\bar{3}} \Phi_3 + m_0 \Phi_0 \Phi_0 \,. \qquad (3.1)$$

We leave aside for now the interactions which will generate the light quark and lepton masses. We assume none of the superpotential interactions are strong, and that at a scale of 10 TeV the SO(7) gauge theory is near a strongly coupled superconformal fixed point. We then break SUSY softly at around 10 TeV by adding mass terms

$$\mathcal{L} \supset M_i^2 |\phi_i|^2 + (\tilde{m}_0^2 \phi_0^2 + \tilde{m}_3^2 \phi_3 \phi_{\bar{3}} + m_\lambda \lambda\lambda + \text{h.c.}) \,. \qquad (3.2)$$

Here ψ_i and ϕ_i are the fermion and scalar components of the superfield Φ_i and λ is the SO(7) gaugino.

We begin with a brief description of the sizes of the above symmetry breaking masses and couplings, delaying a more detailed discussion to a later section. We assume $\mathrm{U}(1)_R$ symmetry breaking masses of the scalars and gauginos are small, as is technically natural. The couplings $h_q, h_{\bar{u}}$ need to be of order one to obtain the top Yukawa. The couplings h'_s and h_s can be of order one and serve to give mass to the fields Y and $\bar{Y}$. The supersymmetric m_3 mass term will play an important role in the dynamics which follows, and is a few TeV. The supersymmetry breaking scalar masses M_i, are of order 10 TeV. The rest of the mass terms are smaller. In order to protect the global symmetry of the little Higgs, m_0 should be of order a GeV, while $\tilde{m}_0$ can be 100 GeV. The term $\tilde{m}_3$ can be anywhere between 10 TeV and 0. The mass of the gaugino, m_λ, should be of order a few TeV. The smaller masses are protected by symmetry and thus there are a variety of mechanisms that could account for their size; we will content ourselves here with the observation that they are natural in the sense of 't Hooft.

We assume that below the scale of the scalar masses, the SO(7) gauge group confines almost immediately due to its large gauge coupling. In what follows we assume that the Standard Model squarks, sleptons and gauginos have a mass of 10-20 TeV or higher (as allowed by naturalness) and participate neither in the low energy dynamics, nor the phenomenology of the model.

3.3 Dynamics and spontaneous symmetry breaking

We are interested in physics of the low energy composite states. Such states can only be composites of the fermions ψ_i and the gaugino λ. Ignoring the weak symmetry breaking interactions from the superpotential and weak gauge interactions, the approximate global symmetries of the theory below 10 TeV are an SU(11) which acts on the 11 fermions in the fundamental representation of SO(7) (the ψ_i fields) and an anomaly free U(1), carried by λ as well as the ψ_i fields. There are no nice solutions to the 'tHooft anomaly matching conditions for SU(11)$\times$U(1), so it is reasonable to expect that part of the global symmetry breaks. The most attractive channel is for a $\lambda\lambda$ condensate, breaking the U(1). It is conceivable that the SU(11) symmetry would be spontaneously broken to SO(11) by a $\psi_i\psi_i$ condensate, but it is also possible to match the 'tHooft conditions in a nice way, with composite spin 1/2 fermions formed of $\psi_i\psi_j\lambda$, in an antisymmetric tensor of SU(11). Note that the anomaly of the antisymmetric tensor of the SU(11) is 7, so such massless bound states match the SU(11) anomaly of the fundamental fermions. It therefore seems plausible, and even reasonable by analogy with supersymmetric gauge dynamics, that the $\psi\psi$ condensate does not form, and the SU(11) remains unbroken.[2]

Note that if there were fewer fermions, the simple composites would not match the anomalies of the global symmetries. Furthermore, the mass term m_3 explicitly breaks the SU(11) symmetry to SU(5) $\times$ SU(3). Some of the composite fermions which are massless in order to match the SU(5) anomalies contain ψ_3 and $\psi_{\bar{3}}$ as constituents. It therefore seems likely that if the mass term m_3 becomes large, the SU(5) chiral symmetry will be spontaneously broken to SO(5), certainly if m_3 were as large as the confinement scale, Λ of 10 TeV this would happen. Once the symmetry is broken, the composite fermions will acquire a mass proportional to m_3 to some power. In particular, we are interested in the masses of the top partners (composites transforming as $(5,3)$ and $(5,\bar{3})$). From (2.11) we see that for a top Yukawa of order one, the symmetry preserving mass of the top partners should be of order f, rather than the larger $4\pi f$ expected from naive dimensional analysis in a strongly coupled theory. However, in our case the mass, besides being proportional to the SU(5) chiral symmetry breaking scale, must also contain the spurion which breaks the SU(6) symmetry of ψ_3 and $\psi_{\bar{3}}$ to SU(3). Thus, it should be proportional to $4\pi f\frac{m_3}{\Lambda}$. We should therefore take the explicit symmetry breaking scale m_3 to be a few TeV. At the same time, we require that the spontaneous symmetry breaking scale, $4\pi f$, should be large, at around 10 TeV, or else the top Yukawa will be too small.[3]

Let us then make an assumption about the dynamics. In strongly coupled supersymmetric theories, one can study analogous effects of explicit symmetry breaking in determining spontaneous symmetry breaking, and the scale of spontaneous breaking can be much larger than the explicit symmetry breaking. This is because the confined supersymmetric theories often have a nontrivial moduli space of vacua, that is, they have massless scalars, and even a small perturbation, if it gives a light or massless scalar a tadpole, can drive a big vev. No such massless scalars are expected for non supersymmetric theories. However, if the number of flavors of a non supersymmetric theory is tuned to some critical num-

[2]The antisymmetric composite fermions do not match discrete anomalies, where the symmetry is a $Z_2 : \psi_i \to -\psi_i$. We thus conjecture that this discrete symmetry is broken by an SU(11) preserving condensate of the form $< \lambda\psi^{11} >$.

[3]Alternatively, it also possible to lower the mass of the top partners by a factor of order a tenth from their naive $4\pi f$ value even if the SU(11) symmetry is broken. It is conceivable that increasing the size and varying the phases of the m_λ, m_3, and $\tilde{m}_3^2$ soft masses (each of which is renormalized by the others via strong SO(7) interactions) decreases the mass of top partners.

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ber which divides a chiral symmetry breaking phase from a non chiral symmetry breaking phase, then turning on a small mass (below the strong coupling scale) for one or more flavors should lead to a phase transition. It is reasonable to assume that this phase transition is of second order. A second order transition necessarily implies a divergent correlation length, i.e., a massless scalar. If, for a near critical number of flavors, the spectrum includes a scalar whose mass is zero for a critical value of some mass parameter, then, even when this mass parameter is not tuned to the critical value, this scalar will be anomalously light. For a theory with a second order phase transition and a near critical number of flavors, a small mass term results in a large vev.

We now consider an toy example of such dynamics, and write down an effective potential for the symmetry breaking order parameters. These order parameters, S_{ij}, have the quantum numbers of $\psi_i\psi_j$ bound states. As we do not expect to have a weakly coupled description of the dynamics above 10 TeV, the following effective potential should only be taken as crude estimation of the size of symmetry breaking order parameters and not as an accurate description of the interactions of $\psi_i\psi_j$ bound states. With naive dimensional analysis and SU(11) global symmetry as guides, we expect the first couple of terms (in an expansion of S^2/Λ^2) of the effective potential to take the following form:

$$\mathcal{L} \supset a_1 m_3 \frac{\Lambda^2}{4\pi} S_{3\bar{3}} + \text{h.c.} + a_2\Lambda^2 \operatorname{tr}(S^\dagger S) + 4\pi a_3 m_3 S_{3i} S^\dagger_{ij} S_{j\bar{3}} + \text{h.c.} + \\ +(4\pi)^2 \frac{a_4}{11} \operatorname{tr}(S^\dagger S S^\dagger S) + (4\pi)^2 \frac{a'_4}{66} \operatorname{tr}(S^\dagger S)^2 + \cdots . \tag{3.3}$$

The first and third terms come from the SU(11) to SU(3) × SU(5) breaking mass term; all other terms are SU(11) symmetric. The peculiar normalization of the last two terms indicates their natural size assuming they induce quantum corrections of order one to the mass term when all the a_i's are of order one. As a consequence of the first term, $S_{3\bar{3}}$ will get a vev of order $a_1 m_3/(4\pi a_2)$.

The order parameter Φ of SU(5) to SO(5) breaking is the five by five submatrix of S_{ij} which corresponds to bilinears of ψ_2, ψ'_2, and ψ_0. The vev $S_{3\bar{3}}$ will induce the following potential for Φ:

$$\mathcal{L} \supset a_2\Lambda^2 \operatorname{tr}(\Phi^\dagger \Phi) + (4\pi)^2 \frac{2a'_4}{11} \left(\frac{a_1 m_3}{4\pi a_2}\right)^2 \operatorname{tr}(\Phi^\dagger\Phi) + \\ +(4\pi)^2 \frac{a_4}{11} \operatorname{tr}(\Phi^\dagger\Phi\Phi^\dagger\Phi) + (4\pi)^2 \frac{a'_4}{66} \operatorname{tr}(\Phi^\dagger\Phi)^2 \cdots . \tag{3.4}$$

For a second order phase transition to occur, a'_4 should be negative, with a_4 positive and slightly larger, so that there is a net positive quartic term. For m_3 larger than a certain critical value, the vev of Φ, f, will be about $\Lambda/(4\pi)$ (assuming a_2 is of order one). We will take f to be about a TeV to match our little Higgs model. Consistent with our assumption that a small mass term can lead to a sizable vev, we take the parameters a_i such that the critical value of m_3 will also be a few TeV. For example, the choice $a_4 = -a'_4 = a_1 = 5$ with all other parameters equal to one, gives an m_3 of 2 TeV. Below 10 TeV, we thus have the desired SU(5)/ SO(5) Nambu-Goldstone bosons, coupled to composite fermions $X_{ij} \sim \psi_i\psi_j\lambda$, which, as a consequence of symmetry breaking, are no longer massless.

Let us now describe the masses and interactions of these fermions. Including the symmetry breaking vevs from above, we find that below 10 TeV, we get the desired SU(5) preserving interaction of the little Higgs top sector as well as couplings of the Nambu-Goldstone modes to the additional fermions:

$$\mathcal{L} \supset b_1 \frac{4\pi m_3}{\Lambda} f X_{5\bar{3}} \Sigma^\dagger X_{53} + b_2 \frac{(4\pi f)^2}{\Lambda} \operatorname{tr}(\Sigma^\dagger X_{55} \Sigma^\dagger X_{55}) + \\ + b_3 \frac{m_3^2}{\Lambda} (X_{3\bar{3}} X_{3\bar{3}} + X_{33} X_{\bar{3}\bar{3}}) + \text{h.c.} \tag{3.5}$$

With a m_3 about a TeV, we have a top Yukawa of order one. For our particular choice for the potential, the vev of $S_{3\bar{3}}$ is similar to f, and thus we find that all composite fermions have comparable masses, parametrically of order m_3^2/Λ. This might seem surprising at first in a vector like theory where mass inequalities [42, 43] imply composites are at least as heavy as their constituents (violated for $m_3 \ll \Lambda$). However, recall that the scalar superpartners of the ψ_i fields, whose masses are of order the strong coupling scale, do not decouple from the dynamics. Since the path integral measure is then not necessarily positive, the mass inequalities do not apply.

Having composite fermions in addition to the necessary top partners, with masses also of order a TeV, could be a common feature of composite little Higgs models. With the mechanism of being near a critical number of flavors, and breaking the symmetry which keeps the fermions degenerate with a small spurion, all fermion masses are generically of the same order. Thus near a TeV, for similar models with SU(11) symmetry, there will be neutral color adjoint fermions, charge $\pm 4/3$ color triplets and anti-triplets, fermions of charge ± 1, a weak triplet, weak doublets of charge $\pm 1/2$, and two singlets.

3.4 The explicit symmetry breaking interactions

Let us now estimate the effect of the various symmetry breaking interactions, making sure that their contributions to the little Higgs mass are acceptable. We will use naive dimensional analysis to approximate the size of operators in the low energy theory. First, consider the M_i^2 soft masses, for the five scalars ϕ_2, ϕ_0, and ϕ_2'. These break the SU(5) $\times$ U(1) symmetry to SU(2)^2 $\times$ U(1)^3 and so can give mass to the little Higgs. We can parameterize them as

$$M^{2\,i}_{\ j} = M^2 \left(\delta^i_j + G^{(2)\,i}_{\ \ \ j} + G^{(2')\,i}_{\ \ \ \ j} \right), \tag{3.6}$$

where M is taken to be near Λ, while $G^{(2)}$ and $G^{(2')}$ are the mass shifts of ϕ_2 and ϕ_2' respectively away from the mass of ϕ_0. The lowest order contribution to the Higgs mass comes from a U(1) preserving operator of the form

$$\Lambda^2 f^2 \operatorname{tr} \left(G^{(2)} \Sigma^\dagger G^{(2')} \Sigma \right). \tag{3.7}$$

Thus there is no danger as long as the mass splittings are of order 1/100. This is quite natural if the scalar masses are degenerate to begin with and acquire splittings due to the symmetry breaking gauge interactions. A possible reason for their initial degeneracy could be that in the UV the masses of the five scalars are much smaller than, for example, the mass of ϕ_3. Then, when the SO(7) coupling becomes strong, the other scalar masses will be additively renormalized in an SU(5) symmetric fashion. Next, consider the U(1) breaking smaller masses. Of these the most stringent constraint is on m_0 which allows for a

$$m_0 \frac{\Lambda^2}{4\pi} f \Sigma_{00} \tag{3.8}$$

term. Consequently m_0 must be around a GeV. A similar operator bounds $\tilde{m}_0$ to be 100 GeV. These terms also provide a mass to the neutral η Nambu-Goldstone boson, and so must be nonzero.

As usual, no gauge or Yukawa interaction single handedly breaks the symmetry protecting the little Higgs, and so a Higgs mass must involve a combination of at least two separate interactions or an interaction with a symmetry breaking mass term. Due to gauge invariance, a gauge coupling spurion can only appear in even powers and since each power comes with a $1/(4\pi)$, combining it with any other interaction (already suppressed by at least a factor of a 100) will lead to a total 10^4 suppression which is sufficient. There is no such constraint on the Yukawa couplings. Indeed, it is possible to have terms with each symmetry breaking Yukawa appearing once, leading to a mere $1/(4\pi)^2$ suppression. However, the Yukawas are charged under Standard Model quark symmetries, and under an $\mathrm{SU}(3)^2$ subgroup of SU(11). Thus any term containing each Yukawa once, must include Standard Model fields and an $\mathrm{SU}(3)^2 \to \mathrm{SU}(3)$ breaking spurion, such as

$$\frac{m_3}{\Lambda} \frac{h_{\bar{u}} h_Q}{4\pi} f Q_3 \Sigma_{02} \bar{u}_3 \,, \tag{3.9}$$

for example (as well as similar terms with $\tilde{m}_3^2$ and $m_\lambda m_3$). These provide a small correction to the top-Higgs interaction and imply a reasonable shift to the little Higgs mass.

The symmetry breaking terms also lead to new interactions for the X_{ij} fermions. Most importantly they provide the necessary SU(5) breaking masses of (2.11):

$$\frac{\Lambda}{4\pi} h_Q Q_3 X_{2\bar{3}} + \frac{\Lambda}{4\pi} h_{\bar{u}} \bar{u}_3 X_{03} \,. \tag{3.10}$$

These have the right size provided that the Yukawa couplings are of order one. In addition, there will be contributions that split the masses of the composite fermions such as

$$\begin{aligned} &\frac{(4\pi f)^2}{\Lambda} \operatorname{tr}(G^{(1,2)} \Sigma^\dagger X \Sigma^\dagger X) \,, \qquad \frac{g^2 f^2}{\Lambda} \operatorname{tr}(Q_i^{a*} \Sigma^\dagger X Q_i^{a*} \Sigma^\dagger X) \,, \\ &\frac{h_Q^2 m_3}{4\pi\Lambda} X_{2\bar{3}} {\Sigma^\dagger}^{2i} X_{i3} \,, \qquad \frac{h_{\bar{u}}^2 m_3}{4\pi\Lambda} X_{0\bar{3}} {\Sigma^\dagger}^{0i} X_{i3} \,, \qquad \ldots \end{aligned} \tag{3.11}$$

all of which are one percent corrections.

Thus, except for small modifications, we indeed recover at low energies the desired little Higgs theory and symmetry breaking spurions. We also find additional fermions besides the top partners whose masses and interactions violate the global symmetry only slightly.

3.5 Obtaining the desired parameters of the UV lagrangian

We would like to briefly discuss some important issues concerning the superpotential and soft masses of the UV lagrangian as the SO(7) theory approaches its superconformal fixed point. First, in order to obtain the right top sector, all Yukawa couplings in the superpotential have to be of order one. It would therefore be ideal if the superpotential couplings of (3.1) were marginal. Near the fixed point, however, the anomalous dimension of the $\Phi_i\Phi_j$ meson is $-3/11$ (up to small corrections due to the weaker interactions). All the Yukawa interactions are thus slightly relevant, and so one must assume that the Yukawa couplings were small in the UV, before the theory began its flow towards the fixed point. These then become of order one near 10 TeV where the conformal symmetry is broken.

The running of the soft scalar masses near the fixed point must also be addressed. For each non-anomalous U(1) symmetry, the corresponding combinations of soft masses (weighed by the U(1) charges of their respective fields) is invariant under the RG flow. On the other hand, various combinations of soft masses, such as the one corresponding to an anomalous U(1), flow to zero [44, 45]. In our case the relevant non-anomalous U(1)s are contained in SU(11), while the under the anomalous U(1) all fields Φ_i have the same charge. Consequently, the sum of all scalar masses runs to zero, while the differences between masses is fixed. There is thus a danger that some of the masses squared will become negative, possibly breaking SO(7). This can be easily avoided if we introduce an additional flavor and let the strong group be SO(8). Then we can take the scalar mass squared of this additional flavor to be a bit below the mass of the rest of the 11 flavors in the UV (a relative factor of two is sufficient). As we flow towards the IR fixed point this scalar will get a negative mass squared and break the gauge group back to SO(7). Meanwhile, the scalars of our 11 flavors will have positive masses, of about a factor of three less than the mass of the extra flavor. This follows from the assumption that all 11 flavors have similar masses, and the requirement that the sum of all 12 masses squared runs to zero.

4. Discrete symmetry and dark matter

Any theory of electroweak symmetry breaking which avoids unacceptable levels of proton decay, must preserve baryon and/or lepton number symmetries to a good approximation. If both are preserved, a Lorentz invariant theory will conserve a $(-1)^{3B+L+2S}$ parity (where S is the spin), which we may call dark matter parity.[4] All Standard Model particles have even parity and therefore the lightest parity odd particle (the LPOP) will be stable against decay. In supersymmetric theories this parity is known as R-parity and the LPOP is the lightest superpartner. However *any* baryon and lepton number conserving theory of TeV physics containing either fermions which do not have odd $3B + L$ or bosons that do have odd $3B+L$ has parity odd particles. The lightest of these is stable. Stable neutral particles

[4] Of course, even in theories with baryon or lepton number violation, such as, e.g. theories with Majorana neutrino masses, conservation of a discrete subgroup of baryon and lepton number can make dark matter parity a good symmetry.

at the weak scale with electroweak interactions are good candidates for dark matter.[5] The MSSM, for example, has scalar baryons, scalar leptons, as well as gauginos and Higgsinos as the non-leptonic and non-baryonic fermions. Of the fermions, ¡a mixture of the Higgsinos and weak gauginos, the neutralino is a favorite for dark matter.

In our case, the interactions with the Standard Model fields require the X_{53} fermions to carry baryon number and so be parity even. However, the X_{55}, X_{33}, and $X_{3\bar{3}}$ fermions are odd under dark matter parity (as are the heavier S_{53} scalars). The lightest of these will be stable. All the X fermions are of comparable mass but gauge and other symmetry breaking interactions should raise the masses of the X_{33}, and $X_{3\bar{3}}$ fermions. A promising dark matter candidate is one of the neutral components of X_{55}. All the components of X_{55} are degenerate to within a few percent. Except for the near-degeneracy, the X_{55} fermions are rather similar to the charginos and neutralinos of SUSY models. The 10 2-component fields of the X_{55} have the quantum numbers of the bino, wino and the two Higgsino doublets (of opposite charges). In addition, there is a charged fermion which is a weak SU(2) singlet. All these fermions get masses from the second term in (3.5) of about a TeV. Once the Higgs gets a vev they mix at the ten percent level and there will be additional one percent mass splitting due to both the Higgs and the above symmetry breaking operators. The LPOP will thus mix strongly with its slightly heavier cousins, which will affect estimates of the relic abundance. Exploring the possibility of the LPOP in this model being the dark matter is very interesting and may place further bounds on the various parameters.

5. Precision electroweak corrections

Precision electroweak observables place important constraints on new physics [46 – 52]. The precision electroweak corrections of the minimal SU(5)/ SO(5) model and several little Higgs variants have been analyzed in refs. [21, 24, 18, 53]. For analysis of a structurally similar model, see [54]. The most important precision electroweak corrections come from dimension 6 operators which arise when integrating out heavy particles at tree level. These are of order v^2/f^2, which is parametrically the same as a one loop minimal Standard Model effect. Corrections of order v^4/f^4, or a loop factor times v^2/f^2, are comparable to two loop Standard Model effects and are mostly smaller than the available experimental precision. An important exception is the one loop correction to the ρ (or T) parameter from the top sector, which is suppressed by both a loop factor and v^2/f^2, but is enhanced by color and fermion multiplicity factors, by relatively large couplings, and a moderately large log.

In order to determine the dimension 6 operators in the effective theory at tree level, it suffices to write out the lagrangian in the limit where the little Higgs vev is turned off, and find the mass eigenstates at tree level. One can then integrate out the heavy particles at tree level by solving their equations of motion to order $1/M^2_{\text{heavy}}$. Only the mass and kinetic terms for the heavy fields and couplings to light particles which are linear in the heavy fields affect the equations of motion to this order. Furthermore, unless there is a heavy-light-light coupling which is of order f, or unless the heavy particle is a fermion, the kinetic terms for

[5] A loosely related parity, with similar consequences for dark matter, is the 'T parity' which has been proposed to eliminate tree level corrections to precision electroweak observables [25].

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the heavy fields may be ignored. It is easy to see in our little Higgs theory the operators relevant for precision electroweak corrections are generated from order f heavy-light-light couplings. These couplings involve the weak isospin and hypercharge currents, the little Higgs, and the third generation quarks. Various corrections by our extension to precision observables such as mass mixing of the weak gauge bosons with additional gauge bosons proportional to the Higgs vev, mixing of light fermions with new fermions with different electroweak quantum numbers proportional to the Higgs vev, and heavy particle exchange between light particles, may all be accounted for by minor alterations of the results in [53].

6. Flavor

Thus far we have only considered the interactions of third generation quarks with our new SO(7) sector. Since there will be mixings between the top and and bottom and new composite fermions, there will be flavor violations for the third generation. However, the lighter generations and the leptons, which do not significantly contribute to the Higgs mass at one loop, do not need to have interactions which preserve any global symmetry. We can therefore imagine coupling them indirectly to the SO(7) sector in the UV theory in a way which does not introduce flavor violation besides the Standard Model Yukawas themselves.

For concreteness, here we will discuss a simple way to generate the terms which give rise to the lighter quark and lepton masses in a renormalizable theory. Namely, we couple the quarks and leptons to weak doublet supermultiplets in the same way as in the MSSM, but give this doublet a large ($\sim 10\,\mathrm{TeV}$) supersymmetric mass and no significant vev. That is, we add to the superpotential (3.1) the terms

$$\mu H_u H_d + \lambda^u_{ij} H_u Q_i \bar{U}_j + \lambda^d_{ij} H_d Q_i \bar{D}_j + \lambda^e_{ij} H_d L_i \bar{E}_j + H_d \Phi_2 \Phi_0 \,, \tag{6.1}$$

and soft supersymmetry breaking scalar mass terms

$$B\mu H_u H_d + \text{h.c.} + \tilde{M}_u^2 |H_u|^2 + \tilde{M}_d^2 |H_d|^2 \,. \tag{6.2}$$

where $\mu, B\mu, \tilde{M}_{u,d}$ are of order the supersymmetry breaking scale, and $H_{u,d}$ transform under $\mathrm{SU}(2) \times \mathrm{U}(1)$ as $(2, \pm 1/2)$. At low energies, we will thus get couplings of the quarks and leptons to the little Higgs

$$y_u^{ij} Q_i f \Sigma_{02} \bar{u}_j + y_d^{ij} Q_i f \Sigma^*_{02} \bar{d}_j + y_e^{ij} E_i f \Sigma^*_{02} \bar{e}_j + \text{h.c.} \tag{6.3}$$

with $y^{u,d,e} \propto \lambda^{u,d,e}$. These terms contain the light quark masses, as well as the small mixings between the third and other generations. Note that only the top has both left handed and right handed components mixing with the composite fermion states. Thus in our model it is naturally heavier than the rest of the fermions. The Yukawa couplings of the down quarks and charged leptons to the little Higgs are generically less than of order $\Lambda^2/(4\pi \tilde{M}^2)$. Additional dangerous operators which violate could violate flavor at low energy, by assumption, must be proportional to the only sources of flavor violation,

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namely the matrices $\lambda^{u,d,e}$, which for light flavors are nearly aligned with the low energy Yukawa coupling matrices. Similarly, the flavor violation in the third generation does not significantly affect the first two due to the tiny mixing angles. Therefore, the model safely satisfies current constraints on flavor changing neutral currents.

As for the SUSY flavor problem, we can take the masses of all the Standard Model superpartners to be 10 TeV or even significantly higher [44] without affecting the naturalness of the soft masses of our SO(7) scalars. They will thus have very small contributions to flavor violating processes.

The only potentially significant flavor violation in our model involves the third generation quarks. The lower component of the quark doublet q_3 is the CKM linear combination of quarks $V_{ti}d_i$, where $i = d, s, b$. Thus operator

$$\bar{q}_3\gamma^\mu q_3\bar{q}_3\gamma_\mu q_3 \tag{6.4}$$

can potentially make a contribution to B_s and B_d mixing which is competitive with the Standard Model while the operator

$$(h^\dagger\tau_a D_\mu h)(\bar{q}_3\tau_a\gamma^\mu q_3) \tag{6.5}$$

gives flavor changing Z couplings to the quarks. In the limit where the upper component of q_3 is purely top, the phases in the new contribution to B_s and B_d mixing and to the Z-$d_i - d_j^\dagger$ coupling are the same as the one generated by the standard model top loops. However, if the upper component of q_3 is not an exact mass eigenstate, new CP violating phases can appear in $B - \bar{B}$ mixing and b quark decay amplitudes, at a level which could be as large as standard model loop effects.

7. Recap and conclusions

We have presented a sequence of natural effective field theories, with no fine-tuning or phenomenological difficulties, describing electroweak symmetry breaking. The underlying theory is a supersymmetric theory, valid to an energy scale which could be as high as the Planck scale. This theory becomes strongly coupled at some high scale, above 10 TeV, and approaches an approximate superconformal fixed point. At 10 TeV, soft supersymmetry breaking drives the theory into a confining phase, with an unbroken approximate SU(11) chiral symmetry and several relatively light composite fermions and scalars, with masses of a few TeV. At 1 TeV, explicit breaking of some of the SU(11) chiral symmetry due to mass terms drives spontaneous breaking of the remaining chiral SU(5) symmetry to an SO(5) subgroup, and all composite fermions become heavy. Most of the resulting pseudo-Goldstone bosons get mass from explicit symmetry breaking at the TeV scale, or are eaten by TeV mass gauge bosons. The notable exception is the little Higgs, a doublet which receives a small, ultraviolet-insensitive negative mass squared from loops in the top quark mass sector. Although this Higgs is a composite particle, it acts like a weakly coupled elementary scalar in the effective theory, driving electroweak symmetry breaking at a naturally low scale.

Our model conserves baryon and lepton numbers. We note that this model, as well as a large class of such beyond the standard model theories, contains a new stable particle, due to a conserved "dark matter parity". All Standard Model particles are even under this parity, however our model contains a plethora of parity odd particles. The lightest parity odd particle, the "LPOP", is likely to be a neutral fermion, which may be a good dark matter candidate.

Precision electroweak corrections can provide important constraints on new theories of electroweak symmetry breaking. We give a general effective field theory analysis of the most important precision electroweak corrections in a large class of theories, and discuss the natural parameter regions in which the model gives an acceptable fit to data.

We have described the model and its phenomena in some detail, not because we expect its particulars to necessarily occur in nature, but rather because we find it an instructive example of natural electroweak superconductivity, which differs phenomenologically from the oft-studied MSSM, and hence offers different insights into possible experimental clues to search for in our quest for a solution to the hierarchy problem. This theory provides an example of dynamical electroweak symmetry breaking, which phenomenologically resembles the minimal standard model below a TeV. At the TeV scale, it distinguishes itself via many new fermions including a good dark matter candidate, a weak triplet of new gauge bosons, and a scalar triplet. These new particles may be expected to have striking signatures at the LHC. However exploring the underlying strong dynamics, or uncovering supersymmetry, requires much higher energies.

A. Generic modifications to the T parameter for theories with additional quarks

The corrections to the rho (or T) parameter, for additional vector-like, charged 2/3 and $-1/3$, isospin doublet quarks are given in Lavoura and Silva [56]. In little Higgs models, and in general models with vector-like fermions, one frequently considers many isospin doublets with various charges. In this section, we generalize Lavoura and Silva's formalism to accommodate weak doublet and singlet quarks with arbitrary charges, and show explicitly how decoupling arises. To begin, we write the weak isospin currents.

$$J^{\mu}_{\rm cc} = \frac{1}{4}\sum_{Q,i,j} \bar{\psi}^{Q}_{i}, \gamma^{\mu}\left[V^{Q}_{L\;ij}(1-\gamma_5) + V^{Q}_{R\;i,j}(1+\gamma_5)\right]\psi^{Q-1}_{j} \tag{A.1}$$

$$J^{\mu}_{3} = \frac{1}{4}\sum_{Q,i,j} \bar{\psi^{Q}}_{i}\,\gamma^{\mu}\left[W^{Q}_{L\;ij}(1-\gamma_5) + W^{Q}_{R\;i,j}(1+\gamma_5)\right]\psi^{Q}_{j}\,. \tag{A.2}$$

Here, ${\psi^{Q}_{(L,R)}}_{j}$ represent left and right handed components respectively of the mass eigenstate quarks of charge Q. In the standard model, $V_L^{2/3}$ is the CKM matrix. However the generalized CKM matrices, V_L^Q and V_R^Q, need not be unitary or square, due to the fact that the mass eigenstates may be mixtures of quarks with different weak charges. The matrices $W^Q_{(L,R)}$, which specify the third component of the weak isospin current in terms of mass eigenstates, are square but not necessarily unitary. These matrices may be found as follows.

Define matrices $X^{(u,d),Q}_{(L,R)\;\;ai}$ such that all weak doublet quarks may be written in terms of mass eigenstates as

$$(\tilde{u},\tilde{d})^Q_{(L,R)_a} = X^{(u,d),Q}_{(L,R)\;\;ai}\,\psi^Q_{(L,R)_i}\,. \tag{A.3}$$

Here the quarks $\tilde{u}^Q_a$ and $\tilde{d}^Q_a$ are not mass eigenstates, but are members of weak doublets with third component of isospin respectively up and down. Unlike in the standard model, weak isospin up and down quarks do not necessarily have charges +2/3 and -1/3. The mass eigenstates are defined such that all masses are real and positive. We continue to label the quarks by electric charge since the mixing matrices can never violate electric charge conservation. The matrices X satisfy the unitarity conditions

$$\sum_i X^{(u,d),Q}_{(L,R)\;\;ai} X^{(u,d),Q\,\dagger}_{(L,R)\;\;ib} = \delta_{ab} \tag{A.4}$$

and

$$\sum_i X^{(u),Q}_{(L,R)_{ai}} X^{(d),Q\,\dagger}_{(L,R)_{ib}} = 0\,. \tag{A.5}$$

Then

$$V^Q_{(L,R)_{ij}} = \sum_a X^{u,Q\;\;\dagger}_{(L,R)_{ia}} X^{d,Q-1}_{(L,R)\;\;aj} \tag{A.6}$$

and

$$W^Q_{(L,R)_{ij}} = \sum_a \left(X^{u,Q\;\;\dagger}_{(L,R)_{ia}} X^{u,Q}_{(L,R)_{aj}} - X^{d,Q\;\;\dagger}_{(L,R)_{ia}} X^{d,Q}_{(L,R)_{aj}} \right). \tag{A.7}$$

Note that

$$W^Q_{(L,R)} = V^Q_{(L,R)} V^{Q\;\;\dagger}_{(L,R)} - V^{Q+1\,\dagger}_{(L,R)} V^{Q+1}_{(L,R)}\,. \tag{A.8}$$

Now the T parameter is given by

$$\begin{aligned} T = \frac{N_c}{16\pi s^2 c^2} \Bigg\{ & \sum_{Q,i,j} \Big[(|V_{L\,ij}^{\,Q}|^2 + |V_{R\,ij}^{\,Q}|^2)\,\theta_+(y_i,y_j) + 2\mathrm{Re}(V_{L\,ij}^{\,Q} V_{R\,ij}^{\,Q})\,\theta_-(y_i,y_j) \Big] - \\ & - \sum_{Q,i<j} \Big[(|W^Q_{L\;ij}|^2 + |W^Q_{R\;ij}|^2)\,\theta_+(y_i,y_j) + 2\mathrm{Re}(W^Q_{L\;ij} W^{Q\dagger}_{R\;ij})\,\theta_-(y_i,y_j) \Big] \Bigg\} \end{aligned} \tag{A.9}$$

where subscript latin indices run over the mass eigenstates, and s^2 and c^2 are $\sin^2\theta_W$ and $\cos^2\theta_W$. The functions θ_+, θ_- and y are defined as

$$\theta_+(y_1,y_2) \equiv y_1 + y_2 - \frac{2y_1y_2}{y_1-y_2}\ln\frac{y_1}{y_2}\,, \tag{A.10}$$

$$\theta_-(y_1,y_2) \equiv 2\sqrt{y_1y_2}\left(\frac{y_1+y_2}{y_1-y_2}\ln\frac{y_1}{y_2} - 2\right), \tag{A.11}$$

$$y_{(1,2)} \equiv \frac{m^2_{(1,2)}}{m_z^2}\,. \tag{A.12}$$

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These formulae can accommodate general additional fermions as long as only weak isospin doublets and singlets are considered, and can easily be modified to allow for other weak representations.

The decoupling of heavy vector-like particles is not immediately obvious from eq. (A.9). Decoupling of heavy particles works when the heavy mass limit does not require strong coupling, and fails when the heavy mass limit requires taking some coupling to be large. Examples of failure of decoupling are the heavy Higgs and/or top mass limit of the Standard Model. In the present case, all the new particles can be made arbitrarily heavy without taking any couplings to be large, although one must fine-tune some parameters.

We now demonstrate decoupling of the one loop corrections to the rho parameter from theories such as ours with heavy vector-like quarks, whose masses can be made arbitrarily large without strong coupling.

In a decoupling theory with heavy weak SU(2) doublets and singlets, the heavy mass eigenstates are nearly vector-like weak eigenstates. All doublets come in nearly degenerate pairs (U,D), with mass splitting no larger than v^2. The mixing matrices V and W are restricted as well. We choose a basis where the matrices X are approximately rows of the unit matrix. The leading correction is of order v/M, with M the mass scale of the heavy quarks. Terms of order v/M are only possible between weak doublets and singlets. To see the effects of these restrictions, we make the following definitions.

Let $n_{_Q}$ be the number of doublets in the weak basis whose $T_3 = +1/2$ components have charge Q. Note that $n_{_{(Q+1)}}$ is then the number of doublets whose lower components have charge Q. The total number of fermions with charge Q is

$$N_Q \equiv n_{_Q} + n_{_{(Q+1)}} + \tilde{n}_{_Q}\,, \tag{A.13}$$

where $\tilde{n}_{_Q}$ is the number of singlets with charge Q. Define

$$\begin{aligned} \delta^Q_{ij} &\equiv \delta_{ij}, \text{if} j \le n_{_Q} \\ &= 0, \text{otherwise}\,, \end{aligned} \tag{A.14}$$

where δ_{ij} is the usual Kronecker delta, and i, j run from 1 to N_Q. In the following we will use a basis where, in the limit $v/M \to 0$, $i \le n_{_{(Q+1)}}$ labels quarks which are are $T_3 = -1/2$, $n_{_{(Q+1)}} < i \le n_{_{(Q+1)}} + n_{_Q}$ labels quarks which are $T_3 = +1/2$, and, for $i > n_{_Q} + n_{_{(Q+1)}}$, the quarks are weak singlets. Now

$$V^Q_{L,R_{ij}} = \delta^Q_{\left(i-n_{_{(Q+1)}}\right)\,j} + \tilde{V}^Q_{ij} + \mathcal{O}(v^2/M^2) \tag{A.15}$$

$$\begin{aligned} W^Q_{L,R_{ij}} &= \delta^Q_{(i-n_{_{(Q+1)}})(j-n_{_{(Q+1)}})} - \delta^{Q+1}_{ij} + \\ &\quad + \tilde{V}^{Q\dagger}_{\left(i-n_{_{(Q+1)}}\right)\,j} + \tilde{V}^Q_{i\,\left(j-n_{_{(Q+1)}}\right)} - \sum_k \left(\tilde{V}^{(Q+1)}_{ik}\delta^{(Q+1)}_{kj} - \delta^{(Q+1)}_{ik}\tilde{V}^{(Q+1)\dagger}_{kj}\right) \\ &\quad + \mathcal{O}\left(\frac{v^2}{M^2}\right). \end{aligned} \tag{A.16}$$

Here $\tilde{V}$ is of order v/M. Note that terms of order v/M transform as doublets under $\mathrm{SU}(2)_w$. SU(2) invariance of the effective theory requires that terms of order v/M must connect doublet quarks with singlet quarks. Therefore $\tilde{V}^Q_{ij} = 0$, if either $i \leq n_{_{(Q+1)}} + n_{_Q}$ and $j \leq n_{_Q} + n_{Q-1}$, or if the inequalities $i > n_{_{(Q+1)}} + n_{_Q}$ and $j > n_{_Q} + n_{Q-1}$ are both satisfied.

Since mass and weak eigenstates coincide up to angles of order v/M, and since weak doublets are nearly degenerate, quark masses satisfy

$$\text{if } i \leq n_{_Q}, \text{ then } m^2_{Q\ (i+n_{_{(Q+1)}})} = m^2_{(Q-1)\ i} + \mathcal{O}(v^2)\,, \tag{A.17}$$

where $m^2_{Q\ i}$ labels the mass of the quark flavor i with charge Q, and $i = 1, 2 \ldots N_Q$. Substituting in eqs. (A.17) and (A.16) into eq. (A.9), we find that the terms of order 1 in the V matrices give contributions proportional to $\theta_\pm(M^2/m_z^2, M^2/m_z^2 + \mathcal{O}(v^2))$, and are of order $v^4/(M^2 m_z^2)$. The terms proportional to $\tilde{V}^2$ are proportional to $(v^2/M^2)(\theta_\pm(M_b^2, M_a^2/m_z^2 + \mathcal{O}(v^2)) - \theta_\pm(M_b^2, M_a^2)$ and are likewise of order $v^4/(M^2 m_z^2)$. Finally, the terms in the mixing matrices which are of order v^2/M^2 give even smaller contributions to T, of order $v^6/(M^4 m_z^2)$.

B. Leading corrections to the T parameter in this model

The full one loop contribution to the ρ, or T parameter in the model receives contributions from all the additional charge 5/3, 2/3 and -1/3 quarks, and is a complicated, unilluminating mess. In order to compute the order v^2/f^2 piece, one can use the following effective field theory treatment. Integrate out the heavy quarks at scale f and generate the effective operators,

$$(h^\dagger D_\mu h)(h^\dagger D^\mu h)\,, \tag{B.1}$$

$$(\bar{t}_R h)\gamma^\mu D_\mu (h^\dagger t_R)\,, \tag{B.2}$$

$$(\bar{q}_3 \tilde{h}^\dagger)\gamma^\mu D_\mu (\tilde{h} q_3)\,. \tag{B.3}$$

The coefficients for operators (B.2), (B.3) are computed at tree level. Operator (B.1) is computed at loop level. Under renormalization group scaling from f to v, including the effects of the one loop top quark Yukawa coupling, these operators mix, and so the operators (B.2), (B.3) will give a log enhanced additive contribution to the coefficient of the operator (B.1). One then computes the effects of the operator (B.1) on the W and Z masses at tree level.

Explicitly, the log enhanced piece turns out to be

$$T = \frac{3}{4\pi s_W^2 c_W^2} \frac{m_t^2}{M_z^2} \frac{v^2}{f^2} \left(s_2^2 c_3^4 \log \frac{b^2}{m_t^2} + 2 s_3^2 c_2^4 \log \frac{a^2}{m_t^2} - 2 s_3^2 \log \frac{(\lambda_1 f)^2}{m_t^2} \right). \tag{B.4}$$

Here,

$$s_2 = \frac{\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}\,, \qquad c_2^2 = 1 - s_2^2\,,$$

$$s_3 = \frac{\lambda_3}{\sqrt{\lambda_1^2 + \lambda_3^2}}\,, \qquad c_3^2 = 1 - s_3^2\,. \tag{B.5}$$

Numerically, this is gives the dominant correction to T and is of order -0.2 for $\lambda_1 = \lambda_2 = \sqrt{6}$,with $\lambda_3 = \sqrt{3}$. Other non-enhanced pieces will give a somewhat smaller contribution.

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